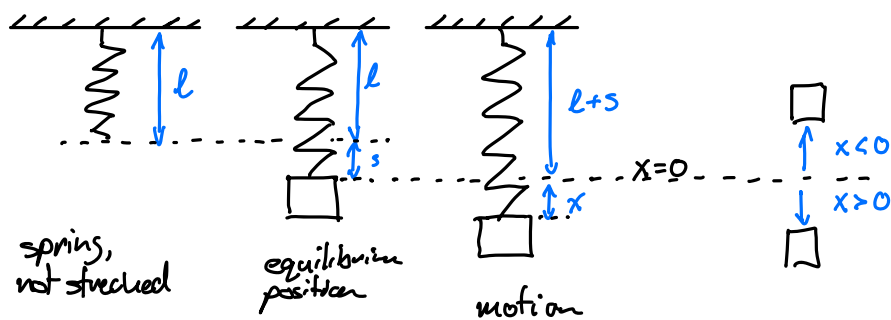


# W3L3 - INTRODUCTION TO FREE UNDAMPED MOTION (SPRING SYSTEM)



Free, undamped  $\rightarrow$  only acted on by spring, no damping effect

$$m \frac{d^2x}{dt^2} + kx = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} + \omega^2 x = 0$$

$x \rightarrow$  displacement       $m \rightarrow$  mass       $k \rightarrow$  spring constant

$\omega^2 = \frac{k}{m}$        $s \rightarrow$  spring elongation

Solve and interpret the IVP. Also find the period and frequency.

$$\frac{d^2x}{dt^2} + 36x = 0, \quad x(0) = 8, \quad x'(0) = 0$$

$$a = 1, \quad b = 0, \quad c = 36$$

$$r^2 + 36 = 0$$

$$\sqrt{r} = \pm \sqrt{36}$$

$$r = \pm 6i$$

$$x(t) = C_1 \cos(6t) + C_2 \sin(6t)$$

$$\omega^2 = 36 \rightarrow \omega = 6$$

$$x(0) = 8 \rightarrow 8 = C_1 \cos(0) + C_2 \sin(0)$$

$$8 = C_1$$

$$x(t) = 8 \cos(6t) + C_2 \sin(6t)$$

$$x'(t) = -48 \sin(6t) + 6C_2 \cos(6t)$$

$$x'(0) = 0 = -48 \sin(0) + 6C_2 \cos(0)$$

$$0 = 6C_2$$

$$C_2 = 0$$

$$x(t) = 8 \cos(6t)$$

$$\text{Period } T = \frac{2\pi}{\omega} \quad \text{Freq } f = \frac{1}{T}$$

$$\omega^2 = 36 \rightarrow \omega = 6$$

$$T = \frac{2\pi}{6} = \frac{\pi}{3} \approx 1.047 \text{ sec}$$

$$f = \frac{6}{2\pi} = \frac{3}{\pi} \approx 0.955 \text{ cycles per second}$$